Solution file HW4

-n 2 i s 10 pt and 2/2/2/4

2. $f(x) = \frac{1}{10}$ for $-5 \le x \le 5$ and $= 0$ otherwise

a. $P(X < 0) = \int_{-5}^{0} \frac{1}{10} dx = .5$.

b. $P(-2.5 < X < 2.5) = \int_{-2.5}^{2.5} \frac{1}{10} dx = .5$.

c. $P(-2 \le X \le 3) = \int_{-2}^{3} \frac{1}{10} dx = .5$.

d. $P(k < X < k+4) = \int_{k}^{k+4} \frac{1}{10} dx = \frac{1}{10} x\Big]_{k}^{k+4} = \frac{1}{10}[(k+4) - k] = .4$.

Q-n 11 is 10 pt and 1/1/1/2/1/1/1/2

11.

a. $P(X \le 1) = F(1) = \dfrac{1^2}{4} = .25$.

b. $P(.5 \le X \le 1) = F(1) - F(.5) = \dfrac{1^2}{4} - \dfrac{.5^2}{4} = .1875$.

c. $P(X > 1.5) = 1 - P(X \le 1.5) = 1 - F(1.5) = 1 - \dfrac{1.5^2}{4} = .4375$.

d. $.5 = F(\tilde{\mu}) = \dfrac{\tilde{\mu}^2}{4} \Rightarrow \tilde{\mu}^2 = 2 \Rightarrow \tilde{\mu} = \sqrt{2} \approx 1.414$.

e. $f(x) = F'(x) = \dfrac{x}{2}$ for $0 \le x < 2$, and $= 0$ otherwise.

f. $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx = \int_{0}^{2} x \cdot \dfrac{x}{2} dx = \dfrac{1}{2}\int_{0}^{2} x^2 dx = \dfrac{x^3}{6}\Big]_{0}^{2} = \dfrac{8}{6} \approx 1.333$.

g. $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_{0}^{2} x^2 \dfrac{x}{2} dx = \dfrac{1}{2}\int_{0}^{2} x^3 dx = \dfrac{x^4}{8}\Big]_{0}^{2} = 2$, so $V(X) = E(X^2) - [E(X)]^2 =$

$2 - \left(\dfrac{8}{6}\right)^2 = \dfrac{8}{36} \approx .222$, and $\sigma_X = \sqrt{.222} = .471$.

h. From **g**, $E(X^2) = 2$.

Q-n 14 is 10 pts and 2/2/2/4. In part a), give 1 pt for the mean and 1 pt for the variance. In part 3, also give 1 pt for each separate question. In part d), give 2 pts for each question.

14.

a. If X is uniformly distributed on the interval from A to B, then $E(X) = \int_A^B x \cdot \frac{1}{B-A} dx = \frac{A+B}{2}$, the midpoint of the interval. Also, $E(X^2) = \frac{A^2 + AB + B^2}{3}$, from which $V(X) = E(X^2) - [E(X)]^2 = \ldots = \frac{(B-A)^2}{12}$.

With $A = 7.5$ and $B = 20$, $E(X) = 13.75$ and $V(X) = 13.02$.

b. From Example 4.6, the complete cdf is $F(x) = \begin{cases} 0 & x < 7.5 \\ \frac{x - 7.5}{12.5} & 7.5 \le x < 20 \\ 1 & 20 \le x \end{cases}$.

c. $P(X \le 10) = F(10) = .200$; $P(10 \le X \le 15) = F(15) - F(10) = .4$.

d. $\sigma = \sqrt{13.02} = 3.61$, so $\mu \pm \sigma = (10.14, 17.36)$. Thus, $P(\mu - \sigma \le X \le \mu + \sigma) = P(10.14 \le X \le 17.36) = F(17.36) - F(10.14) = .5776$.
Similarly, $P(\mu - 2\sigma \le X \le \mu + 2\sigma) = P(6.53 \le X \le 20.97) = 1$.

Q-n 29 is 10 pt and 2/2/2/2/2

29.

a. .9838 is found in the 2.1 row and the .04 column of the standard normal table so $c = 2.14$.

b. $P(0 \le Z \le c) = .291 \Rightarrow \Phi(c) - \Phi(0) = .2910 \Rightarrow \Phi(c) - .5 = .2910 \Rightarrow \Phi(c) = .7910 \Rightarrow$ from the standard normal table, $c = .81$.

c. $P(c \le Z) = .121 \Rightarrow 1 - P(Z < c) = .121 \Rightarrow 1 - \Phi(c) = .121 \Rightarrow \Phi(c) = .879 \Rightarrow c = 1.17$.

d. $P(-c \le Z \le c) = \Phi(c) - \Phi(-c) = \Phi(c) - (1 - \Phi(c)) = 2\Phi(c) - 1 = .668 \Rightarrow \Phi(c) = .834 \Rightarrow c = 0.97$.

e. $P(c \le |Z|) = 1 - P(|Z| < c) = 1 - [\Phi(c) - \Phi(-c)] = 1 - [2\Phi(c) - 1] = 2 - 2\Phi(c) = .016 \Rightarrow \Phi(c) = .992 \Rightarrow c = 2.41$.

Q-n 32 is 10 pt and 2/2/2/2/2

32.

a. $P(X \le 15) = P\left(Z \le \frac{15-15.0}{1.25}\right) = P(Z \le 0) = \Phi(0.00) = .5000.$

b. $P(X \le 17.5) = P\left(Z \le \frac{17.5-15.0}{1.25}\right) = P(Z \le 2) = \Phi(2.00) = .9772.$

c. $P(X \ge 10) = P\left(Z \ge \frac{10-15.0}{1.25}\right) = P(Z \ge -4) = 1 - \Phi(-4.00) = 1 - .0000 = 1.$

d. $P(14 \le X \le 18) = P\left(\frac{14-15.0}{1.25} \le Z \le \frac{18-15.0}{1.25}\right) = P(-.8 \le Z \le 2.4) = \Phi(2.40) - \Phi(-0.80) = .9918 -$
$.2119 = .7799.$

e. $P(|X - 15| \le 3) = P(-3 \le X - 15 \le 3) = P(12 \le X \le 18) = P(-2.4 \le Z \le 2.4) =$
$\Phi(2.40) - \Phi(-2.40) = .9918 - .0082 = .9836.$